Hydrophysical Approach to Soil Detachment and Sediment Transport by Overland Flows

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Abstract: The new equation of soil particles detachment based on hydrophysical approach is suggested. Soil detachment is proportional to the cubed flow velocity if it exceeds the critical value more than 1.6 times. In the field where flow velocity is close to critical value the detachment rate attains stochastic nature and is determined by the probability that instantaneous velocity create the shear stress which exceeds the resistance of soil particles protruded into water. Using a number of detachment tests for adjusting and verification of the model showed a satisfactory results. Coefficients of determination for different sets of data ranged from 0.98 to 0.99. In the field where flow velocity are much less than the critical value the extremely slow detachment of soil particles takes place too. It is probably due to decreasing of cohesion forces under ions exchange between water and soil. It was shown that hydrophysical approach is applicable to modelling of sediment transportation by water flow. The model was justified and verified on the sets of sediment transport data received under the wide range of slope (0.5%—35%) flow depth (0.5 cm — 4 cm) and sediment grain size (0.16 mm — 2.0 mm). The result of verification is good (coefficient of determination equals 0.98—0.99).

1 Introduction

Soil detachment and sediment transportation are the basic components of soil erosion. Thus it is impossible to develop an soil erosion model without a tool which gives an adequate prediction of soil detachment and sediment transport. But as it was shown by Larionov *et al*. (1998) that the most known models of soil detachment do not meet the requirements. The soil detachment by clean water flow are mostly described by deterministic models which can be divided into two groups. The models of the first group (Mirtshulava, 1970; Foster *et al*, 1973) are based on the balance of forces those that kept soil particles in peace and those that try to move them. If the variables of this type of soil detachment equations express trough excess rainfall, slope length and slope gradient it will be clear that this approach can not adequately describe influence of slope factors on soil loss. According to this premise soil loss is function of slope gradient and slope length raised to the two thirds power that contradicts to recent investigation; (Renard *et al*., 1993; Liu *et al*., 1994) which shows that the gradient power magnitude should be one.

The models of the second group are based on the assumption that soil detachment is the function of difference between flow power and its threshold value. Using the simple transformation it is easy to show that in the case soil loss must be straightly proportional to slope length but it is widely known that soil loss is function of slope length raised to power much less than one. Nearing (1991) proposed probabilistic model of soil detachment in order to overcame the discrepancy encompassed in the fact that flow shear stresses are typically are the order of Pa while the soil tensile strength are of the order of kPa. According to this suggestion soil loss should be proportional to slope gradient raised to a tree second power and slope length raised to one thirds power. In the case the sequence does not completely agree with well grounded slope factor equation (Renard *et al*., 1993; Liu *et al*., 1994; Larionov *et al*. 1998).Thus the theoretical sol detachment models partly or in general contradict widely excepted notion about the influence of topographic factors on soil loss.

Analogous situation arose in sediment transport study. The many studies were devoted to the problem, many formulae were developed for the sediment transport capacity prediction nevertheless there is no widely excepted equation for sediment transport prediction. The state of art is conditioned by complexity of the sediment transport phenomenon on the one hand and the extremely wide range of hydraulic parameters should be study on the other hand. It is enough to say that that overland flow bed slopes are one or two orders of magnitude is higher than those encountered in plain rivers. Different premises were used for mathematical description of sediment transport capacity of water flow. Sediment transport was considered as a function of difference flow velocity, shear stress, flow power and their critical values. The probabilistic approach was used too (Einstein, 1950).Consequently the suggested sediment transport equations vary significantly. For example if the most known formulae express as a powered function of flow velocity the power value ranged from 4 up to 6 (Laursen, 1956).The 9 most popular equation have been written in the dimensionless form show the considerable difference too (Alonso *et al*., 1981). Alonso et al. (1981) compared prediction of nine sediment transport formulae with the flume and field data. As expected no formulae satisfactory represented the entire spectrum of sediment and flow characteristics but tree of the tested equations may give satisfactory estimation of transport capacity of different sets of data range, but only the Yalin (1963) formulae can be used to compute sediment capacities for overland flows. Govers (1992) came to nearly the same conclusion. The evaluating of existing transport formulae in overland flow shows that non of the equations yields good prediction over the whole range conditions tested. Promising results can be obtained only for limited number of data or limited range of hydraulic parameters. He considers that a more general sediment transport equations must be developed on the basis of experimental work carried out in a wide range of overland flow condition.

Thus non soil detachment or sediment transport equations can meet the requirements of soil erosion modeling. Here are presented new formulae of soil detachment and sediment transport by overland flow.

2 Soil detachment equation

For the first time hydrophysical approach was used to account for the fact that USLE (1978) severely overestimate the soil loss on steep slopes and gentle long slopes (Larionov,1993). Tree premises was assumed as a basis of the hydrophysical approach to soil erosion. They are:

(1) The detachment and transport of soil particles are represented by work in physical sense, which is accounted for by the active power of water flow.

(2) The detachment of soil particles by flowing water takes place only if a critical level will be exceeded at least by some part of instantaneous flow velocities near the flow bed.

(3) At the point of contact of detached soil particle with flow bed the new particles can not be detached.

The simple mathematical construction shows that according to the first premise soil detachment rate must be proportional to cubed flow velocity. The water flow exerts stress on the soil particles protruded into flow. The stress (F) is proportional to flow velocity raised to the second power

$$F \propto \rho d^2 u^2 \tag{1}$$

where d is the diameter of particle, ρ is the density of water, u is the flow velocity. If the flow velocity exceeds critical meaning soil particle may be detached and the flow impart its velocity which is comparable to that of the flow. The distance (L) which is needed for this may be estimated as

$$L=\frac{1}{2}\ u\ \Delta t \tag{2}$$

where Δt is the time over which the velocity of the detached particles increases from 0 to its maximum. The average stress (P_{evr}) which flow exerts on the soil particle at the Δt time expressed as

$$P_{evr} = \frac{1}{u}\rho d^2 \int_0^u u^2 du = \frac{1}{3}\rho d^2 u^2 \tag{3}$$

Then the work (W) to be done by water flow to impart the soil particle maximum velocity may be written as

$$W=P_{evr} L \propto \rho d^2 u^3 \Delta t \tag{4}$$

Thereafter in the case of shallow flow its power is proportional to average flow velocity raised to the third power then it flows from (4) that the soil detachment is proportional to the power of flow:

$$D_r \propto k_r \gamma u^3 \tag{5}$$

where D_r is the rill detachment rate, k_r is the rill erodibility of soil, γ is the specific weight of water, u is the average flow velocity.

According to the second premise if flow velocity is close to critical value the detachment of soil particles must have an stochastic nature. In the case detachment events depends on overlap of two probability distribution. The first is the distribution of instantaneous flow velocity, the second is the distribution of resistance of soil particles. Probability of events falling within the certain limits is ordinary determined from the special tables but it inconvenient for our case. It easier to use the integrated curve of probability expressed by logistic function. Then the probability (P_v)of instantaneous flow velocity may be approximately described as

$$P_v = \left[1 + 10^{a(1-u/u_0)}\right]^{-1} \tag{6}$$

where u is the average flow velocity, u_0 is the critical velocity, a is the coefficient depended on the instantaneous flow velocity dispersion. According to measurement of instantaneous flow velocity (Mirtshulava, 1967) coefficient a equals approximately 4. In order to convenience the probability the probability of soil particle tension stress should be described by logistic equation with the same variables as in the case of flow velocity. Because of the stress exerted on soil particles by water flow is proportional flow velocity raised to the second power the average relative value of soil tensile strength may be expressed as squared critical flow velocity. Then the probability (P_s) of soil particle tensile strength may be written as where b is the coefficient depended on the dispersion of soil particles tensile strength. Other indications are the same.

$$P_s = \left[1 + 10^{b\left(1-u^2/u_0^2\right)}\right]^{-1} \tag{7}$$

Recalling the sequence flowed from the first premise (Eq.5) the equation of soil detachment (D_r) by clear water should has a form

$$D_r = k_r \gamma \; u^3 \left[1 + 10^{a(1-u/u_0)}\right]^{-1} \left[1 + 10^{b\left(1-u^2/u_0^2\right)}\right]^{-1} \tag{8}$$

The indications are the same.

The simple mathematical construction based on the third premise shows that bed load should decrease detachment capacity of water flow and the influence is strongly effected by soil aggregate stability. The equation 8 of soil detachment by clear water flow was adjusted and verified using an experimental data sets (Kuznetsov, Glazunov, 1985; Nearing et al., 1981; Larionov, krasnov, 1997) obtained under laboratory conditions over the wide range of flow velocities (0.21 m/s —1.97 m/s). Adjusting of the equation (9) showed that it should be revised and written in the form

$$D_r = \gamma \; u_s^3 \left\{ k_{1r} \left[1 + 10^{-4(-u_s/u_0)}\right]^{-1} + k_{2r} \left[1 + 10^{4(1-u_s/u_0)}\right]^{-1} \left[1 + 10^{b\left(1-u_s^2/u_0^2\right)}\right]^{-1} \right\} \tag{9}$$

where u_s is the average flow velocity in the near bed standard layer of 1cm thickness, m/s; k_{1r} and k_{2r} are the rill erodibility one which applied to the velocities that are less than critical value, one which applied to the velocities that are higher than critical value, s^2/m; u_0 is the critical flow velocity in the near bed layer of 1 cm thickness, m/s. Other indications are the same. The coefficients of determination between measured and estimated soil detachment rates for the different sets of data are high (Fig.1).

The flow velocity at the height of roughness elements was used too, but it makes the application of model more complicated. The constant value of coefficient b equaled 6 may be applied for the soils of uniform aggregate size. If the soil of different aggregate size (natural ploughed horizon)the coefficient attain the lover value. The critical velocity can be determined from the graph of soil detachment plotted against the cubed average flow velocity.

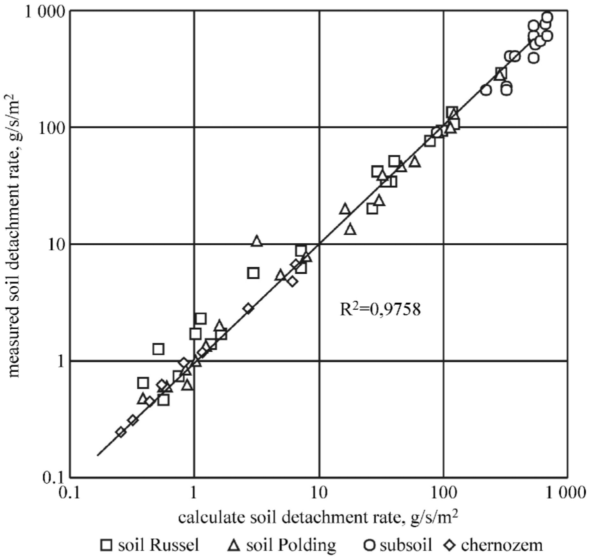

Fig.1 Predicted detachment rates using Eq. 9 vs. measured detachment rate

It can be proposed that the critical velocity is a function of soil tensile strength. The average flow velocity (u_s) in the near bed standard layer of 1cm may be estimated by transformed equation by Izbashas and Haldre (1959) proposed for flows on rough bed

$$u_s = u\, D^{-1,333} \tag{10}$$

where D is the flow depth, cm; u is the average flow velocity, m/s. Equation (9) shows that the soil detachment is governed by probability low only if flow velocity meet the condition $0.4\, u_0 \leqslant u \leqslant 1.6 u_0$. In the opposite cases soil detachment is proportional to the cubed flow velocity. How it comes to agreement with the fact that soil strength is typically on the order of kPa, while the maximum fluctuated shear stresses are of order first hundred Pa (Nearing, 1991a)? Two supposition may dun for explanation of the contradiction. The first one is believed to be due to weakening cohesion forces under influence of ion exchange between the surface layer of soil and water. If the ion exchange is lasting a long time enough the cohesion forces can disappear totally and soil particles can be taken of by water flow as cohesionless material like sand. It is typically to flow velocities which are less than a critical value. For the case the erodibility (k_{1r}) is on tens of s^2/m. If flow velocity is enough to impart to soil particles protruded into water reciprocal movement (trembling) tearing of the soil particles can be resulted from fatigue (Mirtshulava, 1970)which needed much less stress than tensile. In the case erodibility is on hundreds of s^2/m.

The sequence flowed from the third premise that sediment transported as bed load decreases the detachment capacity of flow was proved experimentally. The detachment capacity of flow loaded by coarse particles (D_{r1}) order of a few millimeters in diameter and of 1.0—1.2 g/cm specific weight may be expressed in the form

$$D_{r1} = D_r\, e^{-0.0005X} \tag{11}$$

where X is the quantity of particles transported as bed load above the flow bottom, units/m^2. Other indications are the same. The suspended sediment in concentration as high as 300 g/l did not show any influence on the detachment capacity of water flow.

3 Sediment transport equation

The take of or entreinment particles of cohesionless material by water flow does not essentially differ from the soil detachment and thus the above mentioned approach can be applied to sediment transport if the third premise substitute by the another one. The water flow attains the sediment concentration which corresponds to the transporting capacity of flow if there is no deficiency in sediment. In the case the quantity of settled sediment per unit of bed per unit of time equals to entrained sediment. Obviously that the flow attains the maximum concentration of sediment at the distance equaled to the maximum length of sedimental particle trajectory. Let us divide the length of trajectory on the great number equal segments. Given E, E and S are the quantity of sediment transported over segment ,settled on and entrained from each segment consequently. Let us assume too that K is the part of transported over the segment sediment which settles on the segment. Assuming that no sediment settles on the first segment and that E sediment entrains on each segment it may be written for the first segment $S_1=0$, T=E. On the second segment settling and transporting of sediment may be expressed as

$$S_2 = EK;\ T_2 = E - EK = E + E(1-K) = E[1+(1-K)]$$

Accordingly it may be written for the third and forth segments

$$\begin{aligned} &S_3 = E[1+(1-K)]K; \\ &T_3 = E + E[1+(1-K)] - E[1+(1-K)]K = E + E[1+(1-K)] = E[1+(1-K)+(1-K)^2] \\ &S_4 = E[1+(1-K)+(1-K)^2]K \\ &T_4 = E + E[1+(1-K)+(1-K)^2] + E[1+(1-K)+(1-K)^2]K \\ &\quad = E[1+(1-K)+(1-K)^2+(1-K)^3] \end{aligned}$$

Thus the sediment transport over the *n*-th segment should has a form

$$T_n = E[1+(1-K)+(1-K)^2+(1-K)^3+\cdots+(1-K)^{n-1}]$$

The sum the items in the square brackets equals K^{-1} then the equation can be rewritten in the form:

$$T_n = EK^{-1}$$

Thus the sediment transport is proportional to entrainment of sediment from bottom and inversely proportional to sediment settling. Recalling the sequence flowed from the first and second premises and that sediment may be transported in tree forms (creep, saltation and suspension) the equation of sediment transportation (*T*)by overland flow should has a form

$$\begin{aligned} T = \gamma\, u^3 \{ &k_1[1+10^{a(1-u/u_{01})}]^{-1}[1+10^{b(1-u^2/u_{01}^2)}]^{-1}[1+10^{-a(1-u/u_{02})}]^{-1} \\ &+ k_2[1+10^{a(1-u/u_{02})}]^{-1}[1+10^{b(1-u^2/u_{02}^2)}]^{-1}[1+10^{a(1-u/u_{03})}]^{-1} \\ &+ k_3[1+10^{a(1-u/u_{03})}]^{-1}[1+10^{b(1-u^2/u_{03}^2)}]^{-1}\} \end{aligned} \tag{12}$$

where γ is the specific weight of water, u is the average flow velocity, k_1, k_2, k_3 are the coefficients of sediment transport it the form of creeping, saltation and suspension accordingly, u_{01}, u_{02}, and u_{03} are the critical flow velocity for corresponding type of sediment movement, *a* and *b* are the coefficient which depend on instantaneous flow velocity and resistance of sediment grains to entraining accordingly.

For adjusting of the model the series of flume tests where conducted over the wide range of sediment grain size (0.16 mm—2.0 mm), slope (0.5%—35%) and water depth (5 mm — 40 mm).The highest sediment concentration achieved in tests equals 2.3 kg/l. If sediment concentration achieves 1.5 kg/l the water flow acquires a wave type movement. After adjusting the equation of sediment transport capacity has got a form

$$\begin{aligned} T = \gamma\, u^3 \{ &0.5[1+10^{4(1-u_\Delta/u_{\Delta 01})}]^{-1}[1+10^{6(1-u_\Delta^2/u_{\Delta 01}^2)}]^{-1}[1+10^{-4(1-u_\Delta/u_{\Delta 02})}]^{-1} \\ &+1.37[1+10^{4(1-u_\Delta/u_{\Delta 02})}]^{-1}[1+10^{6(1-u_{\Delta 02}^2/u_{\Delta 02}^2)}]^{-1}[1+10^{-4(1-u\Delta/u_{\Delta 03})}]^{-1} \\ &+2.25[1+10^{4(1-u_\Delta/u_{03})}]^{-1}[1+10^{6(1-u_{\Delta 03}^2/u_{\Delta 03}^2)}]^{-1}[1+10^{-4(1-u_\Delta/u_{\Delta 04}}]^{-1} \\ &+2.54[1+10^{4(1-u_{\Delta\Lambda}/u_{\Delta 04})}]^{-1}[1+10^{6(1-u_\Delta^2/u_{\Delta 04}^2)}]^{-1}\}, \end{aligned} \tag{13}$$

where u_Δ is the flow velocity at the height of roughness element which equaled to 0.7 of average grin size diameter, m/s; $u_{\Delta 01}$, $u_{\Delta 02}$, $u_{\Delta 03}$ and $u_{\Delta 04}$ are the critical flow velocity at the height roughness element for different type of sediment movement including wave movement accordingly, m/s. Other indications are the same. The values of critical velocities ($u_{\Delta 0n}$) can estimated by empirical formula of the form

$$u_{\Delta 0n}=0.2n^{1.1}\,d^{\,0.5} \tag{14}$$

where n is the sequential number of critical velocity, d is the average diameter sediment grain size, mm. The flow velocity at height of roughness element (u_Δ) can be estimated by formulae (Izbash, Haldre, 1959) in the form

$$u_\Delta=1{,}3\;u(\Delta/D)^{1/3} \tag{15}$$

where u is the average flow velocity, m/s; Δ is the height of roughness elements equaled to average sediment grain diameter multiplied by 0,7, m; D is the water depth, m.

Coefficient of determination between measured and estimated transport capacity equals 0,98 (Fig.2).

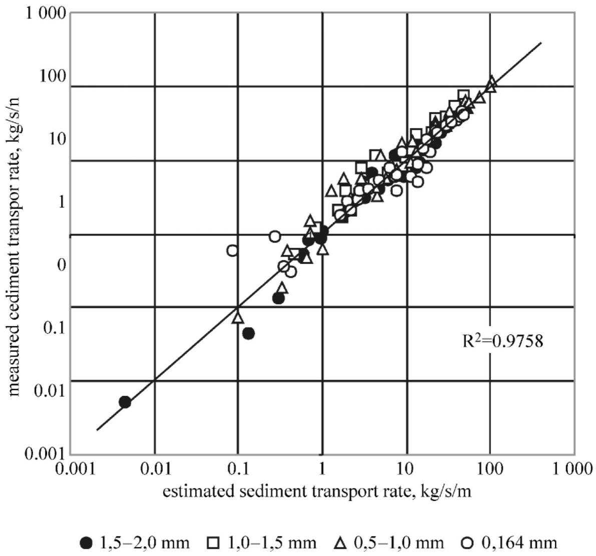

Fig.2 Predicted sediment transport capacity using Eq.13

4 Conclusion

Thus the hydrophysical approach gives an satisfactory description soil detachment and sediment transport over the great range of hydraulic, soil and sediment conditions and the developed equations may be proposed for the physically based soil erosion model.

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