Infiltration into Soil with Dynamic Surface Seals

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Abstract: For the purpose of developing infiltration equation under a dynamically sealling soil surface, Richards' equation of unsaturated flow for a layered system is considered. An analytical approach based on the concept of dynamic wetting front is utilized to form the basis of fundamental solutions of Richards' equation applied to the layered system. Since the interface between the layer and the substrate gives rise to a discontinuity in water content, another dynamic variable representing hydraulic conductance is introduced to facilitate the analysis. Thus, we consider a dynamic seal of thickness h(t) to be on top of the soil column, $z \geq h$. Both media are homogeneous and isotropic and the seal is much less conductive than the column and h is of the order of a few millimeters. First, the general solution of unsaturated flow with appropriate boundary conditions are developed, which consists of concentration and flux boundary value cases. The dynamic condition at the interfaces, between the seal and the substrate permits utilization of these solutions to yield infiltration rate under natural rainfall condition. The same set of solutions also are used to examine the role played by the seal parameters. In particular, infiltration process by the concept of hydrodynamic conductance is examined through the solutions of Richards' equation. The dynamics of cumulative infiltration R is studied at the end by the solution of a set of differential equations in terms of R, wetting front ∗, and the interfacial water content 2_0. An analytical closed for m solution for R is also found for certain cases.
Keywords: infiltration, sealing soils, wet front

1 Introduction

Soil surface sealing is a highly complex and dynamic process affected by soil properties, soil surface conditions and the nature of the hydrologic event. It is a commonly occurring phenomenon on bare and exposed soil which is primarily due to the structural degradation of a thin layer in the soil surface during a rainstorm or irrigation event. The seals are known to appreciably impact soil erosion and runoff processes.

The relationship between soil seal development on one hand and soil properties and rainstorm characteristics on the other hand is not well established. Until recently, the primary focus has been to relate seal development on different soils under simulated rainfall conditions with fixed rainstorm characteristics to subseal matrix potentials (Sharma *et al.*, 1981), to soil erosion parameters (Bradford *et al.*, 1987) or to soil chemical properties (Shainberg and Letey, 1984). However, no index or set of soil properties have been identified that universally describes the soil susceptibility to surface seal development. Besides soil properties, rainstorm characteristics may also affect seal development. Published accounts, in particular those articles concerned with model development, have expressed the effects of seal development on infiltration in terms of changes in the saturated hydraulic conductivity of the sealing zone (Moore, 1981) or in terms of the cumulative rainfall energy (van Doren and Allmaras, 1978; Brakensiek and Rawls, 1983). Experimental studies (Römkens *et al.*, 1985, and Römkens *et al.*, 1986) based on rain infiltration into laboratory prepared soil columns with different rainstorm intensities, have suggested that seal development depends on both the cumulative rainfall energy and the rainfall pressure potential and infiltration on carefully prepared soil columns (Römkens *et al.*, 1990).

We consider a crust (seal) of thickness h to be on the top of the soil column, $z \geq 0$. Both media are homogeneous and isotropic and the crust (seal) is much less conductive than the column. Since one of the objectives of this investigation is to gain insight into the effect of a seal on infiltration under natural conditions of rainfall, it will be assumed that h is quite small (say a few millimeters). We will assume that the rainfall rate is given by r(t) which is a non-negative function of time t. The water content distribution is expected to assume a shape as shown in Fig. 1, where 2_0(t) is the interface water content, 2_1(t) and 2(t) are the water contents pertaining to the crust and the column, respectively. The wetting front z = ∗(t) has entered the soil column (∗≥0). The arrival time of the wetting front at the interface (z = 0) is quite small and the analysis presented here primarily focuses on the case when ∗>>h.

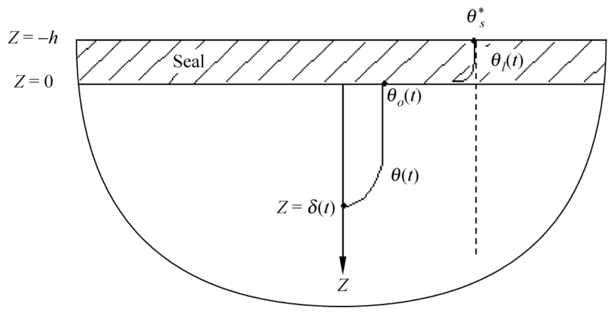

Fig. 1 A schematic representation of the water content in a 2-layered soil profile with a less hydraulically conductive surface seal or crust

The non-linearity of the Richards' equation has often dictated the employment of numerical solution techniques in solving soil water flow problems. The use of these techniques has been stimulated and facilitated by the availability of high speed digital computers. The solutions are then displayed by means of graphs and numerical tables. However, these solutions often lack the generality that is attributable to closed-form, semi-analytical, or approximate parametric solutions. Moreover, important features of interaction processes such a seal formation, remain hidden in the numerical approach. For this reason an analytical approach of using a spectral series solution for Richards' equation has been recently developed (Römkens and Prasad, 1992). This approach is further utilized in this paper to address the infiltration process through layered soil in a manner which will be useful for the study of the dynamics of seal formation due to raindrop impact.

Rain infiltration to homogeneous, isotropic soils is governed by the Richards' equation which for one dimensional, vertical flow is given by:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D(\theta) \ \frac{\partial \theta}{\partial z} \right) - \frac{K(\theta)}{\partial z} \tag{1}$$

where 2 is the soil water content, *z* is the vertical coordinate, *t* is the time coordinate, and *D* and *K* are the concentration dependent diffusivity and hydraulic conductivity functions, respectively.

The diffusivity function D(2) has been the subject of numerous theoretical and experimental studies in the past. Two important aspects which have particular significance in our analysis must be emphasized here. These two aspects relate to the behavior of the diffusivity function for a fixed, rigid soil matrix when 2 |0 and 2 |0_s, where 2_s is the saturated soil water content. Near the wetting front 2 = 0, the diffusivity *D* approaches zero whereas near saturation it tends to become infinite. A representation which retains these features has been studied by Ahuja and Swartzendruber (1972) and has been previously utilized in other studies (Prasad *et al.*, 1998; Römkens and Prasad, 1992). The diffusivity functions for the crust and the column are D_1 and *D*, respectively whereas the conductivity functions are represented by K_1 and *K*. Both sets of functions depend on the water content 2 and in particular the diffusivity function for the soil column is given by:

$$D(\theta) = \frac{a\theta^n}{(\theta_s - \theta)^{n/5}} \tag{2}$$

where a (. 1—12) and *n* (. 3—4.5) are constants which values are obtained upon fitting the diffusivity values to the soil water content (Ahuja and Swartzendruber, 1972).

2 Dynamic equation of cumulative infiltration *R*

The mathematical analysis is based upon an expansion solution of the water content 2 near the wetting front *z*=∗. The water content is given by the expression:

$$\theta = \eta^{\alpha} f(\eta(t)); \quad \eta = \delta - z \tag{3}$$

in which

$$\alpha = \frac{1}{n}; \quad 0 < \alpha < 1 \tag{4}$$

and

$$f(\eta;t) = \sum_{m-0}^{\infty} a_m \eta^m \quad ; \quad a_m = a_m(t) \tag{5}$$

The governing equation of the growth of the wetting front ∗ is given by: (Römkens and Prasad, 1992; eqs 10-13b)

$$\frac{d\delta}{dt} = C_0 a_0^n; \quad C_0 = \frac{a}{n} \theta_s^{n/5} \tag{6}$$

where θ_s(t) is the saturation water content of the substrate soil. Further, the water content at the interface of the substrate soil, θ_0 is given by:

$$\theta_0(t) = a_0 \delta^{\alpha} + a_1 \delta^{a+1} + a_2 \delta^{a+2} + \cdots \tag{7}$$

Let

$$A_k = a_k \delta^{a+k}; \quad k = 0,1,2,\cdots \tag{8}$$

so that from (7) we have:

$$A_0 = \theta_0 - A_1 - A_2 - \cdots \tag{9}$$

Substitution of Equation (7) and (9) in equation (6) yields:

$$\frac{\mathrm{d}\delta^2}{\mathrm{d}t} = 2C_0(\theta_0 - A_1 - A_2 \cdots)^n \tag{10}$$

The flux *q* may be calculated from

$$q(z,t) = D\left(\frac{\alpha}{\eta}\theta + \eta^{\alpha} \sum_{m-0}^{\infty} m\, a_m \eta^{m-1}\right) + K \tag{11}$$

which at the interface leads to the flux entering the substrate given by

$$q_0 = \frac{a}{\delta}(\alpha\theta_0 + A_1 + A_2 + \cdots)\frac{\theta_0^n}{(\theta_s - \theta_0)^{n/5}} + K_0 \tag{12}$$

In the above $K_o = K(\theta_0)$ is the hydraulic conductivity of the substrate material at the seal interface. The integration of the water content profile yields after some simplification the following relationship:

$$\frac{A_0}{\alpha+1} + \frac{A_1}{\alpha+2} + \cdots = \frac{R(t)}{\delta} \tag{13}$$

where $R(t)$ is the cumulative infiltrated amount in the substrate. The above equation (13) may be rearranged by utilizing (9) into the following expression:

$$A_1 = (\alpha+2)\theta_0 - \frac{R}{\delta}(\alpha+1)(\alpha+2) - \frac{2A_2(\alpha+2)}{\alpha+3} \tag{14}$$

so that equation (10) may be written as

$$\frac{\mathrm{d}\delta^2}{\mathrm{d}t} = 2C_0\left[\frac{R}{\delta}(\alpha+1)(\alpha+2) - \theta_0(\alpha+1) + \frac{A_2(\alpha+1)}{\alpha+3}\right]^n \tag{15}$$

Equation (15) provides the basis for investigating the cumulative infiltration dynamics as impacted by seal development. For this purpose, note that (15) contains primarily three dynamic quantities R, $*$ and θ_o. The unknown quantities A_2, A_3, etc. play minor role in the interaction processes and their effect may be incorporated by means of a shape factor such that other boundary conditions are fulfilled. Further, from equation (12) we obtain

$$q_0 - k_0 = \frac{C_0 n}{\delta}(\alpha\theta_0 + A_1 + 2A_2 + \cdots)\frac{\theta_0^n}{\left(1 - \frac{\theta_0}{\theta_s}\right)^{n/5}} \tag{16}$$

For the purpose of a clear understanding of the physics of the role played by the seal on the infiltration process, we approximate the system of equation (15) and (16) by the following relationships:

$$\frac{\mathrm{d}\delta^2}{\mathrm{d}t} = 2C_0\left[\frac{R}{\delta}(\alpha+1)(\alpha+2) - \theta_0(\alpha+1)\right]^n \tag{17}$$

$$q_0 - k_0 = \frac{C_0 n}{\delta}(\alpha+1)\left[2\theta_0 - \frac{R}{\delta}(\alpha+2)\right]\theta_0^n \tag{18}$$

We also obtain from (13):

$$\frac{R}{\delta} = \frac{\theta_0}{\alpha+1} \tag{19}$$

In view of (19), equations (17) and (18) simplify into:

$$\frac{\mathrm{d}\delta^2}{\mathrm{d}t} = 2C_0\theta_0^n \tag{20}$$

$$q_0 - k_0 = \frac{C_0}{\delta}\theta_0^{n+1} \tag{21}$$

The system of equations (19-21) permit us now to examine various seal models and their impact on cumulative infiltration R.

3 Effect of seal formation on *R*

From a hydraulic standpoint, under natural rainfall-runoff conditions, the presence of surface seals impeded the flow of water into the substrate soils. The seal induced: (i) increased saturation, (ii) reduced the capillary flow magnitude, and (iii) clog the voids with smaller particles, which then further reduces the matrix flow. The diffusivity of the surface materials becomes negligible, but since the suction at the bottom of the seal may be high, the diffusivity there is significant. The magnitude of the capillary flow $D\frac{\partial\theta}{\partial z}$ in the seal may be replaced by

$D(\theta_s - \theta)/h$ where 2 is the water content at the bottom of the seals. Thus a hydraulic model of the flow in the seal is proposed here in which the flux q_s out of the seal into the substrate is given by

$$q_s = k_s^* - k^*\theta_0 \tag{22}$$

In the above, it is assumed that k_s^* includes the values of the saturated hydraulic conductivity and

$$k_s^* > k^*$$

Both k_s^* and k^* may vary with the water content. Since the seal thickness is very small, the amount of water stored in the seal is also small. Thus the total infiltrated amount into soil is adequately represented by R, so that the infiltration r is given by

$$r = \frac{\mathrm{d}R}{\mathrm{d}t} \tag{23}$$

We may also assume that the infiltration rate r is the same as the flux q_o into the substrate soil column.

Also, note that the infiltration rate r is the same as q_s. By combining (19), (22) and (23) we obtain

$$\frac{\mathrm{d}R}{\mathrm{d}t} = k_s^* - k^*\theta_0 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\theta_0\delta}{\alpha+1}\right) \tag{24}$$

so that we have the following system of equations describing the present interaction problem

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\theta_0\delta}{\alpha+1}\right) = k_s^* - k^*\theta_0 \tag{25}$$

$$\frac{\mathrm{d}\delta^2}{\mathrm{d}t} = C_0\theta_0^n \tag{26}$$

Relationships (25) and (26) are is a coupled set of ordinary, nonlinear differential equations, whose numerical integration appears straight forward. At the present time, however, we find an interactive procedure starting with an integration of (20) which leads to

$$\delta^2 = 2C_0\int_0^t \theta_0^n(\tau)\,\mathrm{d}\tau \tag{27}$$

As an approximation, we assume

$$\delta = \sqrt{2C_0\theta_0^{\frac{n}{2}}}\sqrt{t} \tag{28}$$

so that from (24) we obtain

$$\frac{\mathrm{d}R}{\mathrm{d}t} = k_s^* - k^*\chi\frac{R^{\frac{2}{n+2}}}{t^{\frac{1}{n+2}}} \tag{29}$$

where

$$\chi = \frac{(\alpha+1)^{\frac{2}{2+n}}}{(2C_0)^{\frac{1}{2+n}}} \tag{30}$$

The following approximate solution for the cumulative infiltration R is obtained

$$R = k_s t - k^*\beta^*\left(\frac{n+1}{n+2}\right)t^{\frac{n+2}{n+1}} \tag{31}$$

where

$$\beta^* = \chi(k_s)^{\frac{2}{n+2}} \tag{32}$$

The response given by (31) is depicted in Fig. 2 for the purpose of comparison.

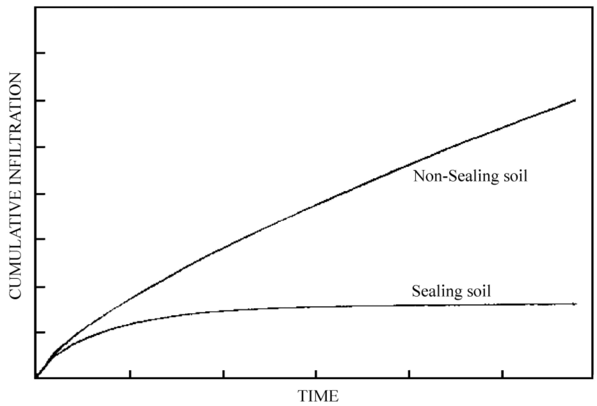

Fig. 2 A schematic representation of the cumulative infiltration for sealing and non-sealing soil profile conditions

4 Seal impedance model

Another model which represent seal effect may be quantified in terms of a flow impedance to infiltrating water. Such an expression may be obtained by rearranging Darcy's equation

$$Z = \frac{1}{\beta_c} = \frac{\Delta H}{r} \tag{33}$$

where Z is the hydraulic impedance, r is the flux (infiltration rate), and $\exists_c$ is the hydraulic conductance (Römkens *et al*., 1986), and $H = H_{20} - H_{10}$ is the difference in the hydraulic potential across the sealing zone with H_{10} being the hydraulic potential at the air-soil surface interface and H_{20} being the hydraulic potential at the seal-bulk soil interface. In most studies of rain infiltration into sealing soils, the total impedance or conductance through the sealing zone is the primary property of interest. Thus parametric results of $\exists_c$ for various cases were recently reported by Römkens *et al.* ,(1990b). It is apparent from these results that

$$\beta_c = \beta_c(E, i) \tag{34}$$

where E is the cumulative rainfall energy and i is the rainfall intensity. For a given rainfall intensity, the values of $\exists_c$ decrease very rapidly with the cumulative rainfall energy E. Note that for this case, the cumulative rainfall energy E is a function of time, $E(t)$. Further, the soil water potential beneath the seal, H_{20} is a function of the water content 2_0 and may be given by

$$H_{20} = p_1 \theta_0^{p_2} \tag{35}$$

where p_1 and p_2 are constants. For instantaneous ponding on soils with developing seals Eq. (33) becomes:

$$r = H_{20} B_c \tag{36a}$$

In the following we develop a closed form approximation for the cumulative infiltration:

$$R = \int_0^y r(\tau)\,\mathrm{d}\tau \tag{36b}$$

which is based on the results of the previous section. Thus, we have

$$r = \frac{\mathrm{d}R}{\mathrm{d}t} = p_1 \beta_c(E, i)\,\theta_0^{p_0} \tag{37}$$

so that following substitution of Eq. [19] into Eq. [37], one obtains:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = p_1 \beta_c(E, i)\left(\frac{R}{\chi_1 \delta}\right)^{p_2} \tag{38}$$

Eq. [38] is the desired result which may now be integrated by iteration. A schematic representation of the infiltration relationship [38] is shown in Fig.2 for a hypothetical soil under both sealing and non-sealing conditions. This relationship is very similar to those measured by Römkens *et al.*, (1990 a, Fig. 5-14) on an Atwood soil subjected to simulated rainstorm of constant rainfall intensity.

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