The Cusp-Catastrophe Model of Landslide Caused by Irrigation

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Abstract: According to the cusp-catastrophe theory of nonlinear science, the water-percentage of soil were regarded as the state variable, and angle of internal friction and clay grain percentage of soil were regarded as the bound variable in cusp-catastrophe model. Based on the standard equation of the cusp-catastrophe theory, the relation formula among water-percentage of soil, angle of internal friction and clay grain percentage of soil has been derived by used coordinate transform. Field data were used to verify this formula, that the results calculated from the cusp-catastrophe formula agree well with field data. The relation of ground water and irrigation water had analyzed, and the means of control landslide was been studied from control the quantity of irrigation water. The purpose of this paper is to explore the means of control landslide from the viewpoint of nonlinear science.

Keywords: catastrophe theory, cusp-catastrophe, irrigation, landslide, water-percentage of soil

1 Introduction

Along with the development of nonlinear science, the nonlinear science is applied the studies the landslide in recent studies. The catastrophe model of the constitutive relation of soil was build by He Guanne and Zhang Yemin in 1991. The catastrophe model of platform and inclined top of soil were used to describing the catastrophe constitutive property of soil in landslide by Gao peng in 1994. Water is the key factor in landslide. The ground water and rainwater are taking into account in currently studies, and the irrigation water is less considered.

Wang Lanming (2000) pointed along with the water-percentage of soil increased in irrigation, the irrigation water was a hidden danger of landslide in loess area. Wang Jiading and Hui Yanghe (2001) had analysis the factor in Heifangtai loess landslide induced by irrigation water. Reference [6] had get large numbers of landslide investigation in loess plateau irrigation area in Gansu Province. Based on the prior studies, the state variable and the bound variable in cusp-catastrophe model of loess landslide crows induced by irrigation water were analyzed in this paper. According to the standard equation of cusp-catastrophe model, the relation formula among water-percentage of soil, angle of internal friction and clay grain percentage of soil has been derived by used coordinate transform. A new way for studying the cusp-catastrophe model of loess landslide crows induced by irrigation water will be discussed.

2 Decided the state variable and bound variable

Water is the key factor in influencing the landslide of soil. The physics and geometry characteristic of soil also affect the landslide. The landslide of soil induced by irrigation water is the result of rainwater, ground water and irrigation water together. In the case of rainwater and ground water are constant, the quantity of irrigation water is the key factor for landslide. Reference [6] pointed the irrigation water translated into ground water effect the landslide. So we can take the water-percentage of soil as the state variable in the cusp-catastrophe model of loess landslide induced by irrigation water.

The physics characteristic of soil that affected the landslide is water-percentage of soil, angle of internal friction, cohesive force and soil unit weight. The water-percentage of soil is the variable in the cusp-catastrophe model, and soil unit weight is not changed in great quantity in the same area. At same times, angle of internal friction and cohesive force has correlation attested by Fan Mingqiao in 1997. So the angle of internal friction or cohesive force can be choose as the variable in the aspect of physics characteristic of soil. Here we can choose the tangent of angle of internal friction as the control variable in the cusp-catastrophe model of loess landslide.

The grain component and size are the variable reflected the geometry characteristic of soil. Cui Peng (1993) took the clay grain percentage of soil as control variable in mud flow incipient. Here we can take the clay grain percentage of soil as other control variable. So we take the water-percentage of soil as the state variable, the tangent of angle of internal friction and the clay grain percentage of soil as two control variables in the cusp-catastrophe model of loess landslide induced by irrigation water.

3 The cusp-catastrophe model of incipient motion of non-uniform sediment

Based on studies above, the state variable in the cusp-catastrophe model of loess landslide induced by irrigation water is the water-percentage of soil (W%), and control variables are the tangent of angle of internal friction $\tan(\varphi)$ and the clay grain percentage of soil (C%). According to the catastrophe theory, the catastrophe model is shown as Figure1. The area in Figure.1 is divided into three sections. The lower section is the area of motionless state of soil. The upper section is the area of stabilization state after landslide, and the middle section is state of landslide unsteadiness. The landslide of soil is the transition process from lower section to upper section.

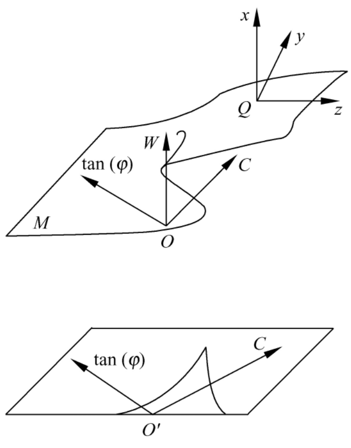

Fig. 1 The catastrophe model of landslide

According to the nonlinear theory (Yi Chuixiang, 1984), the cusp-catastrophe model equation is expressed as follows

$$\frac{\partial\left(\frac{x^4}{4}+\frac{yx^2}{2}+zx\right)}{\partial x}=0 \tag{1}$$

Such as

$$x^3+xy-z=0 \tag{2}$$

The standard equation of the curve in the plane could be written as

$$4y^3 + 27z^2 = 0 \tag{3}$$

The coordinate system *Q* (*x*, *y*, *z*) change into *O* (*W*, *C*, tan(*φ*)), and this express as follows

$$\begin{aligned} x &= l_1 W + l_2 C + l_3 \tan(\varphi) - \alpha \\ y &= m_1 W + m_2 C + m_3 \tan(\varphi) - \beta \\ z &= n_1 W + n_2 C + n_3 \tan(\varphi) - \gamma \end{aligned} \tag{4}$$

Here *α*, *β* and *γ* are *Q* point value in the coordinate system *O* (*W*, *C*, tan(*φ*)), The m_i, m_i, n_i (i=1,2,3) are the direction cosine from the coordinate system *Q*(*x*, *y*, *z*) to the coordinate system *O*(*W*, *C*, tan(*φ*)).

Owing to the *W* coordinate is parallel to the x coordinate, so

$$l_1 = \cos 0° = 1 \tag{5}$$

Then the equation (3) in the coordinate system *O* (*W*, *C*, tan(*φ*)) is rewritten as follows

$$\begin{gathered} (W + l_2 C + l_3 \tan(\varphi) - \alpha)^3 + (m_1 W + m_2 C + m_3 \tan(\varphi) - \beta)(W + l_2 C + l_3 \tan(\varphi) - \alpha) \\ - (n_1 W + n_2 C + n_3 \tan(\varphi) - \gamma) = 0 \end{gathered} \tag{6}$$

The append condition of coordinate transform is expressed as follows

$$\begin{aligned} l_1^2 + m_1^2 + n_1^2 &= 1 \\ l_2^2 + m_2^2 + n_2^2 &= 1 \\ l_3^2 + m_3^2 + n_3^2 &= 1 \end{aligned} \tag{7}$$

and

$$\begin{aligned} m_1 m_2 + n_1 n_2 + l_1 l_2 &= 1 \\ m_2 m_3 + n_2 n_3 + l_2 l_3 &= 1 \\ m_3 m_1 + n_3 n_1 + l_3 l_1 &= 1 \end{aligned} \tag{8}$$

When the coordinate system is right hand coordinate system, then

$$\begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} = 1 \tag{9}$$

Because the curve surface space of loess landslide pass through the coordinate system origin, so

$$f(0,0,0) = 0 \tag{10}$$

From formula (8) and formula (10), we could get that

$$\alpha^3 - \alpha\beta - \gamma = 0 \tag{11}$$

Then

$$\begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} \quad (12)$$

Here $\theta = \text{arctg}\alpha$

Placing formula (12) into (6), we can obtain the formula as follows

$$(W-\alpha)^3 + (W\cos\theta + \tan(\varphi)\sin\theta - \beta)(W-\alpha) - (-C\sin\theta + \tan(\varphi)\cos\theta - \gamma) = 0 \quad (13)$$

Placing formula (11) and ($\theta = \text{arctg}\alpha$ into 13), then

$$(W-\alpha)^3 + \alpha^3 + W(C-\beta')\cos\theta - \tan(\varphi)(\sec\theta - W\sin\theta) = 0 \quad (14)$$

Here $\beta' = \beta/\cos\theta$, α is the eigenvalue of W in the studied area (Yi Chuixiang, 1984). According to the definition of the limit equilibrium stability of soil, α is 18% in the Heifangtai loess landslide area. γ is the eigenvalue of the tangent of angle of internal friction. According to the studies of Cheng Ziyong (1994), the tan(φ) is 0.1 when the landslide is take place. Then placing the α, β, $\beta' = \beta/\cos\theta$, $\theta = \text{arctg}\alpha$ and $\alpha^3 - \alpha\beta - \gamma = 0$ into formula (14), The formula (14) is rewritten as follows

$$(W-0.1824)^3 + 0.98W(C-0.53) - \tan(\varphi)(1-0.178W) + 0.006 = 0 \quad (15)$$

Here W is the water-percentage of soil, C is the water-percentage of soil, φ is the tangent of angle of internal friction.

4 Verifying and application of the formula

Heifangtai platform is fourth terrace of the Yellow river valley. The area of Heifangtai platform is about 13.44 km^2. Since irrigation in 1968, the landslide took place in serially, and it formed the landslide crows in this area. There are about 40 varying sizes landslide along the 10 km long edge of the Heifangtai platform. According to the practical survey, average subsidence quantity in the platform is 1.86 m. There are 15 times landslides happen from 1968 to 1983, it was average 1 times per year, and the distance was 20m—50m. There are 9 times landslides happen from 1984 to 1989, it was average 1.5 times per year, and the distance was 50m—100m. There are 22 times landslides happen from 1990 to 1995, it was average 4.4 times per year, and the distance was 250m—400m. There are 9 times landslides happen from 1996 to 1998, it was average 3 times per year, and the distance was 300m—400m. So along with the seep of the irrigation, the landslide is happen seriously. In the process of irrigation water is seeping, the angle of internal friction and cohesion of soil are reduced along with the increase of water-percentage of soil. When the soil state become from semi-dryness to saturation, the cohesion of soil is reduce from 0.9mPa to 0mPa, and the angle of internal friction is reduce from 40° to 5.7°. So the irrigation water is key factor in landslide of Heifangtai platform. We had used the formula (15) to landslide in Heifangtai platform area as follows.

The formula (15) is verified by the investigate date (Cheng Ziyong, 1994) shown as Table.1. The data computed by formula (15) is the calculate value of the left term of formula (15), it should be 0. It is clear that the computed value by formula (15) is basic corresponds with the investigated data. This indicates that the cusp-catastrophe formula is reasonable, and the results fully reflect the characteristics of loess landslide induced by irrigation water. But there are some errors between the computed data and investigated date. The errors cause by two aspect of the investigated data and computed data. The investigated date in this paper is the peacetime measure data of the landslide area in Heifangtai platform area, and it not value in time of landslide take place. In other aspect, many factors effect soil landslide and the relation among factors are not completely reflected by the cusp-catastrophe formula in which have only three variables. So the cusp-catastrophe model are recognized to be partial and incomplete, and further work will be discussed. The purpose of this paper is to explore the law of loess landslide induced by irrigation water from the viewpoint of nonlinear science.

Table 1 Comparison between formula and field data

Measure pot	W(%)	C(%)	φ(°)	Calculate data
Jiaojia village 1	6.22	84	31.36	–0.52
Jiaojia village 2	8.34	84	32.77	–0.51
Jiaojia village 3	15.51	84	31.38	–0.37
Jiaojia village 4	20.8	84	22.25	–0.1
Jiaojia village 5	22.66	84	26.11	–0.15
Jiaojia village 6	2.71	84	29.12	–0.51
Huangci village 1	14.88	72	25.38	–0.27
Huangci village 2	14.04	72	45.5	–0.81
Huangci village 3	1.12	72	36.27	–0.72
Saleishan	18.8	74	23.3	–0.17
Putaoyuan	21.23	54	21.18	–0.14
Tianshui 1	14.2	74	31	–0.4
Tianshui 2	7.5	74	28	–0.42

The landslide induced by irrigation water has many effect factors. When the rainfall and the nature condition are certain, the irrigation water is key factor in landslide. The quantity of irrigation water decides the seep water quantity, and this influence the water-percentage of soil. According to studies above, there are 352mm irrigation water replenish into ground water in Heifangtai platform area, and the ground water level increase 0.55m per year. Along with ground water level is increasing, the landslide is taking place frequently. So the irrigation water is the primary influence factor to the soil landslide in Heifangtai platform area. At same time, we can use the water-percentage of soil computed by formula (15) to forecast the landslide, and we can control the landslides from control the irrigation water. There are many means of control the quantity of irrigation water, such as technology of save on water irrigation, technology of ground water irrigation return to land and other agriculture engineering technology el.

5 Conclusion

The factor of irrigation water effect to landslide had been analysis in this paper. The water-percentage of soil was took as the state variable, the tangent of angle of internal friction and the clay grain percentage of soil were took as two control variables in cusp-catastrophe model of loess landslide induced by irrigation water. According to the standard equation of cusp-catastrophe model, the relation formula among water-percentage of soil, angle of internal friction and clay grain percentage of soil has been derived by used coordinate transform. A cusp-catastrophe model of loess landslide has been built. The formula (15) is verified by investigate date, and the value computed by formula (19) is basic corresponds with the investigate data. A new way for studying the cusp-catastrophe model of loess landslide crows induced by irrigation water will be discussed.

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