Analytic Determination of Soil Erodibility and Critical Shear Stress

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Abstract: An analytic method for determining the soil erodibility and critical shear stress τ_c of concentrated flow was advanced in the paper. Manipulating the functional relation of sediment concentration with rill length and the analytic method for detachment rate determination using the regressed functional relation with data from simulated erosion experiments, the relationship amongst maximum net detachment rate, soil erodibility and shear stress of the flowing water was derived from this function，and hence the soil erodibility and critical shear stress τ_c were determined. The results showed that soil erodibility is the same for the experimental soil of similar soil physical conditions under different slope gradients. Soil erodibility of silt-clay soil (Loess soil) is 0.3211 ± 0.0003 kg • N^{-1} • s^{-1} on average. Critical shear stress τ_c increases with the slope gradient, namely 3.191 N • m^{-2}, 3.937 N • m^{-2}, 4.127 N • m^{-2}, 4.376 N • m^{-2}, 4.624 N • m^{-2} under 5°, 10°, 15°, 20°, 25° respectively. The soil erodibility and critical shear stress τ_c thus computed were compared with those directly estimated on the basis of experimental data to verify the feasibility of this method. The paper advanced a quick and convenient yet feasible method to calculate the soil erodibility and critical shear stress τ_c.

Keywords: soil erodibility k; critical shear stress; soil erosion prediction model; analytic method

Soil erosion, one of the calamities around the world, has been severely threatening the development of agriculture and society. It is not only a major factor responsible for the degradation of land quality in a long run, but also a major source of non-point water pollution. Therefore, more and more attention has been paid to soil erosion prevention and control. Soil erosion research, soil loss prediction and application of soil erosion control technology should be considered systematically. Soil erosion prediction model is a group of mathematical functions based on the understanding of soil erosion mechanism and processes. As a tool to quantitatively estimate soil erosion intensity, soil erosion prediction model is regarded as a foundation for land use and soil conservation planning. Soil conservation planning in any countries or regions should be based on reliable data of flow velocity and soil loss, and thus the higher prediction precision of model to predict quantitatively the temporal and spatial distribution of soil erosion is required. Since 1980s, scientists started to develop process-based erosion models, such as Water Erosion Prediction Project (WEPP), in which erodibility and soil critical shear stress τ_c are experiential values because of the limitation of experimental and/or analytical method. Other process-based models, such as CREAMS, ANSWERS, EUROSEM, etc, can simulate the soil erosion distribution, but quantitative parameters in these models have been the limitation for model improvements.

Soil erodibility k and critical shear stress τ_c, as important indexes of soil properties, are two of the most important parameters in process-based model such as WEEP, CREAMS, ANSWERS *et al*. To measure and calculate the value of k and τ_c is vital to better prediction of soil erosion with process-based model. Soil erodibility k means the erosive feasibility of soil. Being a comprehensive parameter, it by now cannot be measured directly, but be evaluated with some soil physical property indexes under controlled conditions. Therefore, it is difficult to give a quantitative definition of soil erodibility. Soil critical shear stress τ_c is an important index to characterize the soil mechanical properties which could be

used to analyze quantitatively the occurrence of soil erosion on upland. τ_c is related with soil cohesion and soil inner friction. Soil particles could be detached only when flow shear stress exceeds soil critical shear stress. τ_c is often measured by instruments in fields. Lot of samples are needed to get τ_c value and the measuring precision is limited by time and soil conditions. τ_c in process-based models is calculated by function used in natural river.

An analytic method for determining the soil erodibility and the soil critical shear stress τ_c of concentrated flow is advanced in the paper. The function of sediment concentration with rill length and the analytic expression of the detachment rate are manipulated to determine the soil erodibility and critical shear stress τ_c.

1   Theory

WEPP, based on the physical processes of water and sediment movement, describes soil erosion process and sediment transportation in hill slope with mathematical functions. In order to predict precisely, parameters in the model should have specific physical meaning, and should be measured by experiment directly. The soil erodibility and the soil critical shear stress τ_c are important parameters in WEPP. Sediment yielding function in WEPP is as follows:

$$D_r = k(\tau - \tau_c)\left(1 - \frac{qc}{T_c}\right) \tag{1}$$

where k is rill erodibility (s/m); τ is shear stress of flowing water, acting on the soil particle (Pa); τ_c is critical shear stress of the soil (Pa); qc is sediment load G (kg/m s); T_c is sediment transport capacity of water flow (kg/m s).

When the sediment concentration in flow is zero, the detachment rate approaches its maximum. Under this condition, Eq.1 is reduced to:

$$D_{r\max} = k(\tau - \tau_c) \tag{2}$$

or

$$D_{r\max} = k\tau - k\tau_c \tag{3}$$

Sediment comes from detachment of soil by flowing water. The detachment rate is defined as amount (in kg) of soil detached from a unit area (m^2) in a unit time (s). A steady-state sediment continuity equation based on mass balance is used to describe the sediment balance:

$$\frac{\partial(cq)}{\partial x} + \frac{\partial(ch)}{\partial t} = 0 \tag{4}$$

c(kg/m^3) —sediment concentration, q(m^2/s)—flow rate per unit width, x(m) —down slope distance，h(m) —depth of flow，t(s) —time. As for the unit area, $\frac{\partial(ch)}{\partial t}$ is detachment rate (D_r). Under a given initial and boundary conditions, an analytic method for determining detachment rate of concentrated flow in eroding rills on steep slope was advanced by Lei T.W and Zhang Q.W et al.:

$$D_r = \lim \frac{\Delta c}{\Delta x} \cdot \frac{Q}{w} = \frac{\mathrm{d}c}{\mathrm{d}x} q \tag{5}$$

The following theorem presents the concept equivalently. The detachment rate (in a rill) is the change rate, with respect to rill length, of sediment concentration in the flow rate of unit rill width. Or equivalently, the soil detachment rate is the change ratio of sediment yield with respect to distance times the flow rate of unit rill width. Proving of analytic function is seen as reference [13].

Relationship of the sediment concentration and rill length was advanced in reference [13]:

$$c = A(1 - \mathrm{e}^{-\beta x}) \tag{6}$$

Manipulating Eq.6 and Eq. 5, we have:

$$D_r = q\beta A e^{-\beta x} \tag{7}$$

D_r has its maximum value when x approaches to 0, and thus we have,

$$D_{r\max} = q\beta A \tag{8}$$

$$q\beta A = k\tau - k\tau_c \tag{9}$$

Soil erosion process is the combination of a series of interaction processes between soil erodibility and erosive force. Flow on upland has a certain velocity and energy. Moving on the surface of land, it imposes a shear stress on the soil body. The net soil detachment occurs only when shear stress of water exceeds the critical shear stress of the soil.

Provided the velocity of flow at an infinitesimal cross section of dA is u, and the flow rate across dA is:

$$dQ = udA \tag{10}$$

If the average velocity of section is denoted as v, and the flow rate at cross section of A can be expressed as follows:

$$Q = \int_Q dQ = \int_A udA = vA \tag{11}$$

According to the principle of flow dynamics, shear stress equals to the part of gravity along the flow direction. In order to simplify calculation, cross section of water flow is assumed to be rectangle, and then shear stress could be expressed as:

$$\tau = \gamma sh = \gamma s \frac{Q}{vw} \tag{12}$$

where γ is specific gravity of water, 9,800 N • m^{-3}; s is hydraulic slope, $\sin(\alpha)$, α is slope in degrees; h is depth of flow(m)，Q is flow rate(m^3 • s^{-1})，v is averaged velocity(m • s^{-1})，w is the width of flow(m)。

τ is calculated with Eq.（12），$D_{r\max}$ is calculated with Eq. (8), while k and τ_c are estimated by regressing Eq. (9). Soil erodibility k and soil critical shear stress τ_c are obtained accordingly.

2 Calculation and results analysis

The simulated experiments of rill erosion in laboratory were designed based on the conditions of the occurrence of rill in field. The three treatments were: slope grade, slope length and flow rate. Five slopes (5°, 10°, 15°, 20°, 25°), 8 to 9 slope lengths (0.5 m, 1.0 m, 2.0 m, 3.0 m, 4.0 m, 5.0 m, 6.0 m, 7.0 m, 8.0 m), and three flow rates (2 L/min, 4 L/min, 8 L/min; i.e., 0.12 m^3/h, 0.24 m^3/h, 0.48 m^3/h) were used. Three replicates were made for a total of 405 experiments. We used a silt-clay (loess) soil, typical of the Loess Plateau. Relationship between sediment concentration and rill lengths and corresponding parameters were got from the experimental data. The maximum net detachment rats $D_{r\max}$ were calculated using Eq. 8, and flow shear stress τ were calculated using Eq. 12. The relationship between $D_{r\max}$ and flow shear stress τ are seen in Fig. 1.

From the figures, we can see that the slopes of the lines in Fig. 1 are soil erodibility k. The steeper the slope of a line is, the bigger the k is. The value of flow shear stress could be thought as the critical shear stress of the soil τ_c when the net detachment rate is 0. That means the ratio of the slope of a line to the soil erodibility k is critical shear stress τ_c. Fig. 1 shows that the slopes of lines are almost the same under different slope gradients while the intercepts of different slope gradients are different. Those mean that the erodibilities for different slope are the same and the critical shear stresses are not. Soil erodibility k and soil critical shear stress τ_c are listed in Table 1.
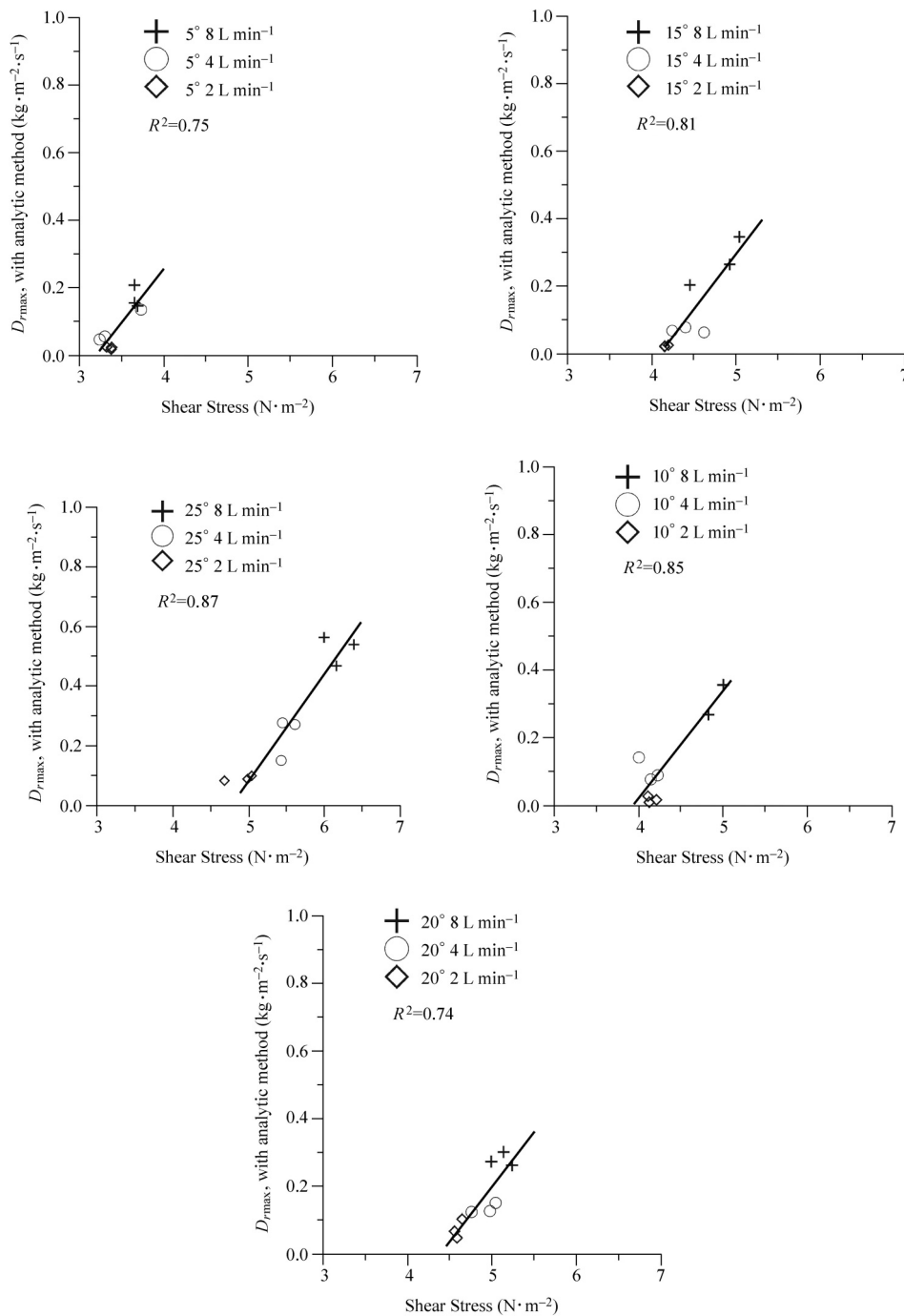
Fig.1 Relationship between the net potential detachment and shear stress

In general, k decreases with increase in soil clay content and soil organic matter content. There have been studies showing that k of soil in field increases with increase in slope gradient. The change in k under different slope gradients did not show in our experiments. From Table 1 we can see that k under different slope gradients are nearly the same. The silt-clay soil was used in this case, in which sandy particle (>0.05mm) is 20.175%, silt particle (0.05mm—0.005mm) 20.175%, and clay particle (<0.005mm) 15.92% and its median size 0.029 mm，and physical clay particle 23.88%. The averaged soil erodibility parameter k is (0.3211 ± 0.0003) kg $\cdot$ N^{-1} $\cdot$ s^{-1}. From the k estimated from the experiment data, we can see the soil used is liable to erosion, and has high susceptivity to erosive agent and transportation. The

determination coefficients R^2, of regressing the data with Eq. (9) under different slopes are greater than 0.74, which means the prediction results can explain more than 74% that corresponding to experimental data. There are 9 samples used in each treatment, and replicate sample is 27.

Table 1 k and τ_c under different slope gradient with analytic method

Slope gradient	k（kg・N^{-1}・s^{-1}）	T_c（N・m^{-2}）	n	Regressed function	R^2
5°	0.3209	3.191	9	$Y=0.3209x–1.0241$	0.75
10°	0.3211	3.937	9	$Y=0.3211x–1.2641$	0.85
15°	0.321	4.127	9	$Y=0.321x–1.3248$	0.81
20°	0.3213	4.376	9	$Y=0.3213x–1.406$	0.74
25°	0.3214	4.624	9	$Y=0.3214x–1.4985$	0.87

What should be noted is that soil erodibility is a physical characteristic of soil. Although k is related to slope gradient and flow rate, it doesn't change remarkably with these two factors according to the theoretical analysis given by Liu [6].

From Table 1 the critical shear stress of soil τ_c increases with the slope gradient, namely 3.191 N・m^{-2}, 3.937 N・m^{-2}, 4.127 N・m^{-2}, 4.376 N・m^{-2}, 4.624 N・m^{-2} under 5°,10°,15°,20°,25° respectively. The coefficients of determination R^2 are more than 0.74 under different hydraulic conditions. When the shear stress exceeds the critical shear that soil is eroded, soil structure would be deformed and destroyed. When slope is lower than 45°, the shear stress on soil would increase with increase of slope gradient, because the vertical force and the force along the slope increase with slope gradient. And thus interface between soil particles increases, and distance between particles decreases accordingly. As a result, both the gravitation and friction between particles increase. Therefore, the friction to prevent shear stress increases in order to maintain the stability of soil body. This explains why critical shear stress increases with increase of slope gradient. At the same time, soil particles are detached easily on steeper slopes when flow shear stress is over the critical shear stress, and so the detachment rate increases with slope.

Comparison of soil erodibility and critical shear stress computed by the analytic method with those estimated directly from experimental data was made and seen in Fig. 2.
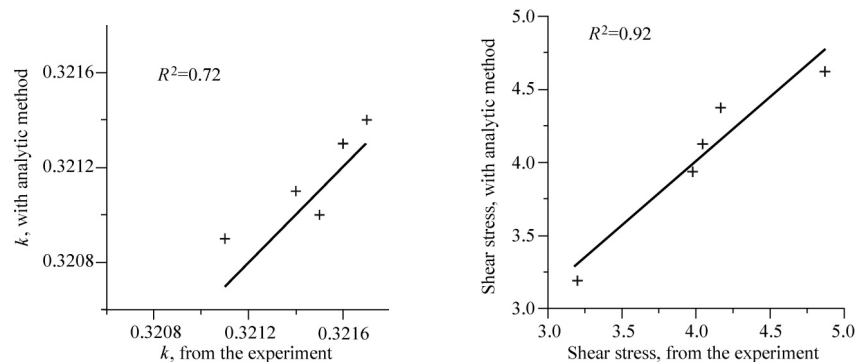
Fig.2   Comparison of soil erodibility and critical shear stress computed by the analytic method with those estimated directly from experiment data

The closer the dots in the Figure to the oblique line, the better the correlation between analytic method and experiment means. From Figure 2, we can see that correlation of soil erodibility parameter and critical shear stress computed by the analytic method with those estimated directly from experimental data is very good. The correlative function of soil erodibility parameter got by analytic method and those estimated with experimental date is $Y = 0.792,439 \times X + 0.066,402,5$，The correlative function of soil

critical shear stress from analytic method and those estimated with experimental data is $Y = 0.880601 \times X + 0.484212$. The coefficients of determination R^2 of k are greater than 0.72 under different hydraulic conditions, and the coefficients of determination R^2 for critical shear stress are more than 0.92.

3   Conclusion

Analytic method to determine soil erodibiltiy k and critical shear stress τ_c was advanced in the paper. k and τ_c from this method have a specific physical meaning because they presents the resistive extent of soil to detachment force. And we can also see from the calculated results that susceptivity of k and τ_c to the hydraulic conditions are quite different; it would be better to consider them conjointly when to develop the soil prediction model. A simple and convenient method to determine k and τ_c was advanced in the paper to improve the precision of soil prediction model.

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