Sediment Transport in Shallow Overland Flow

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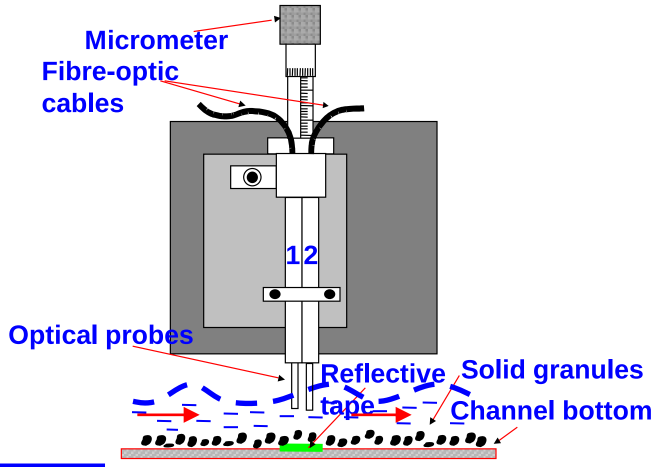
Overview

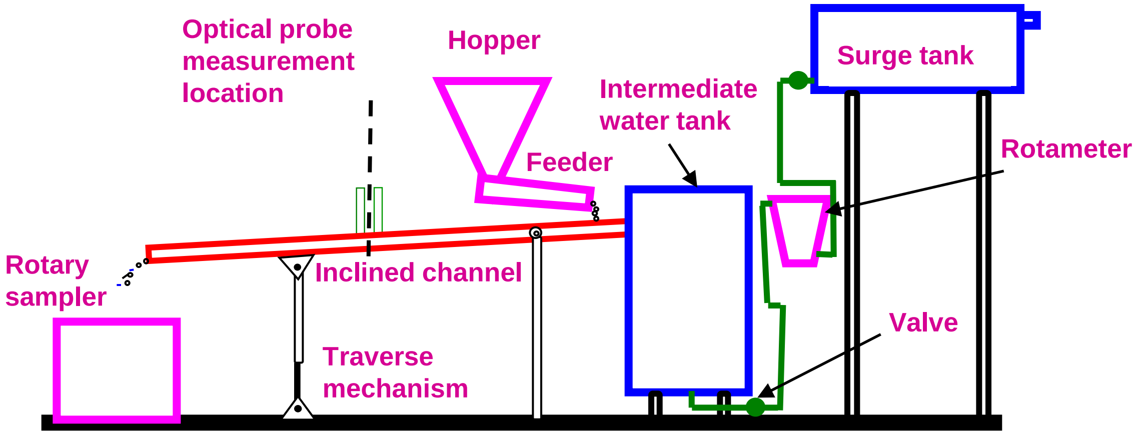
1. Introduction
2. Synopsis of in-house sediment movement research at the grain level.
3. Drag Reduction Analysis
4. Conclusion

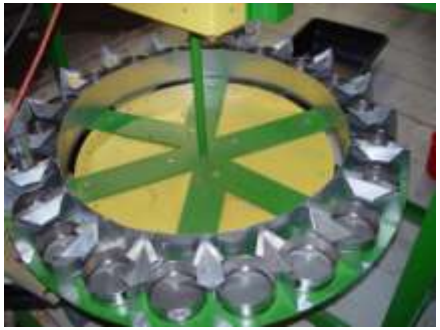
Introduction

- **Numerous Sediment transport studies in channel flow – mostly coarse size material and subcritical flow.**
- **Relative few sediment transport studies in shallow overland flow – mostly aggregated material and super-critical flow.**

Shallow overland flow - Experimental

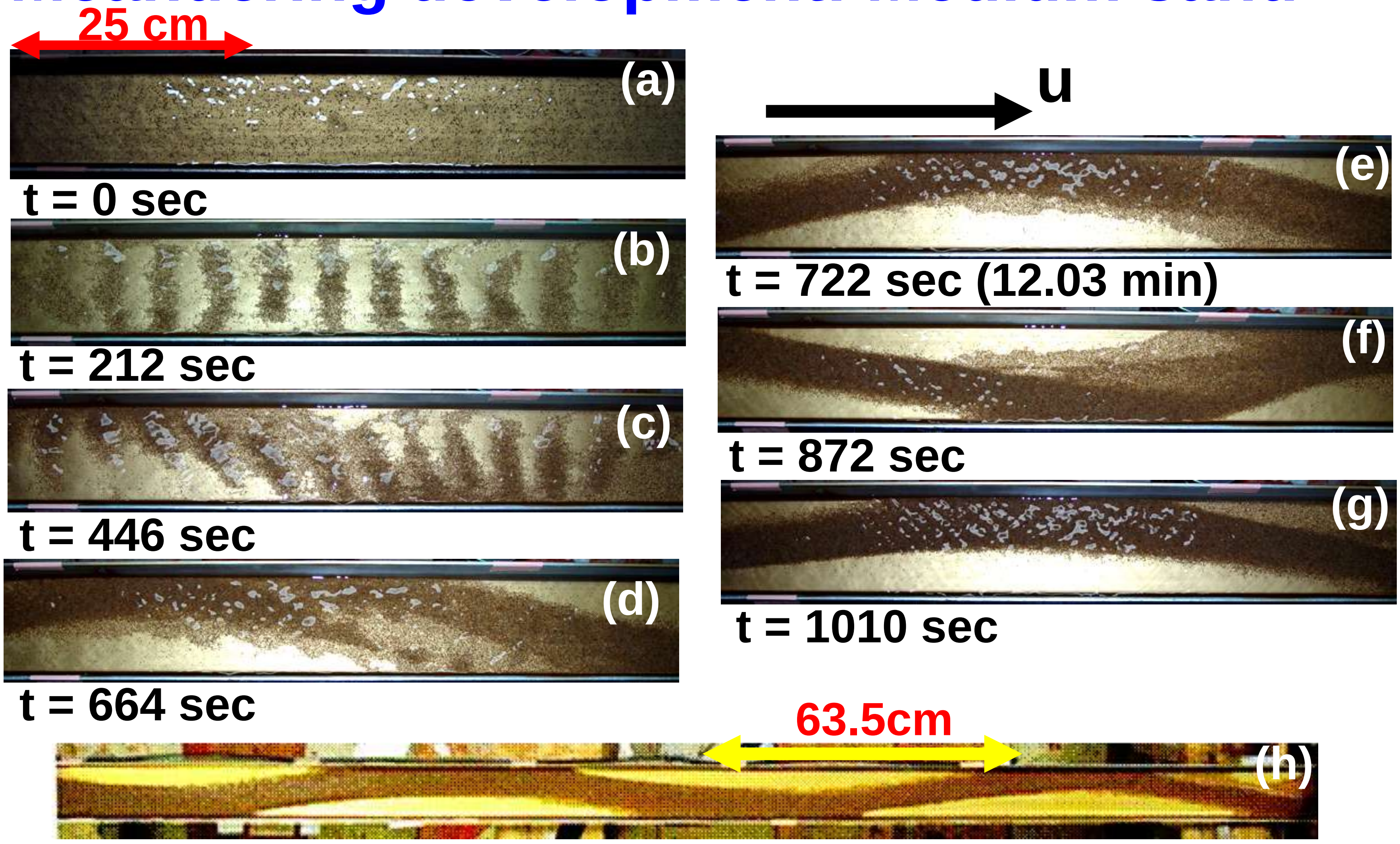






Schematic Diagram of Experimental Set-up

Meandering development: Medium sand



Figs. (a) – (g): Meander formation and its development (all the figures correspond to the same scale (camera placed at a fixed location). Particle size, d_s = 600 – 850 µm. Water flow rate, q_l = 15.7 l/min (Fr_l = 1.45); solids feed rate is constant, m_s = 168.9 g/min. Fig. (h) – Fully developed meander structure (m_t = 10.2 g/min).

Observations

Transport modes

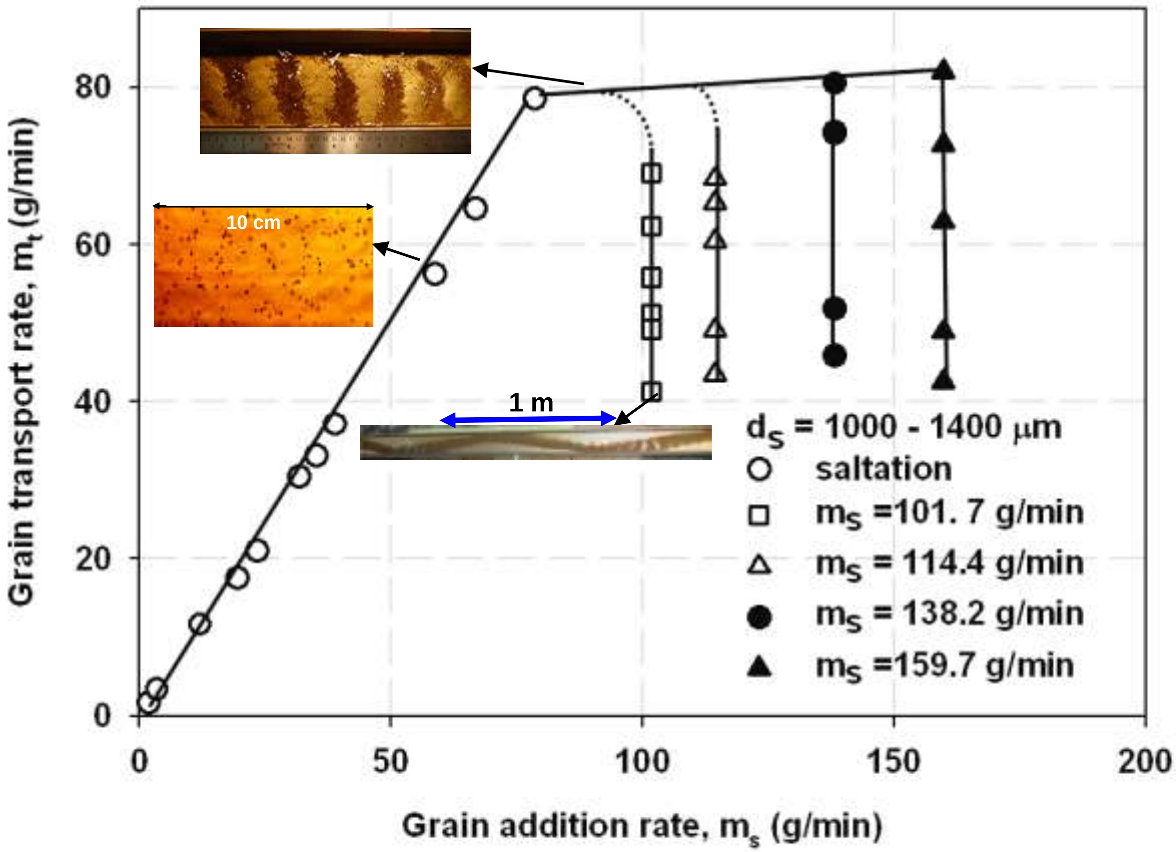


Figure : Sediment transport rates in relation to sediment addition rates. Coarse sand (d_s = 1000-1400 µm) water flow rate q_l = 15.7 l/min (Fr_l = 1.45)

Transport rate vs solid concentration

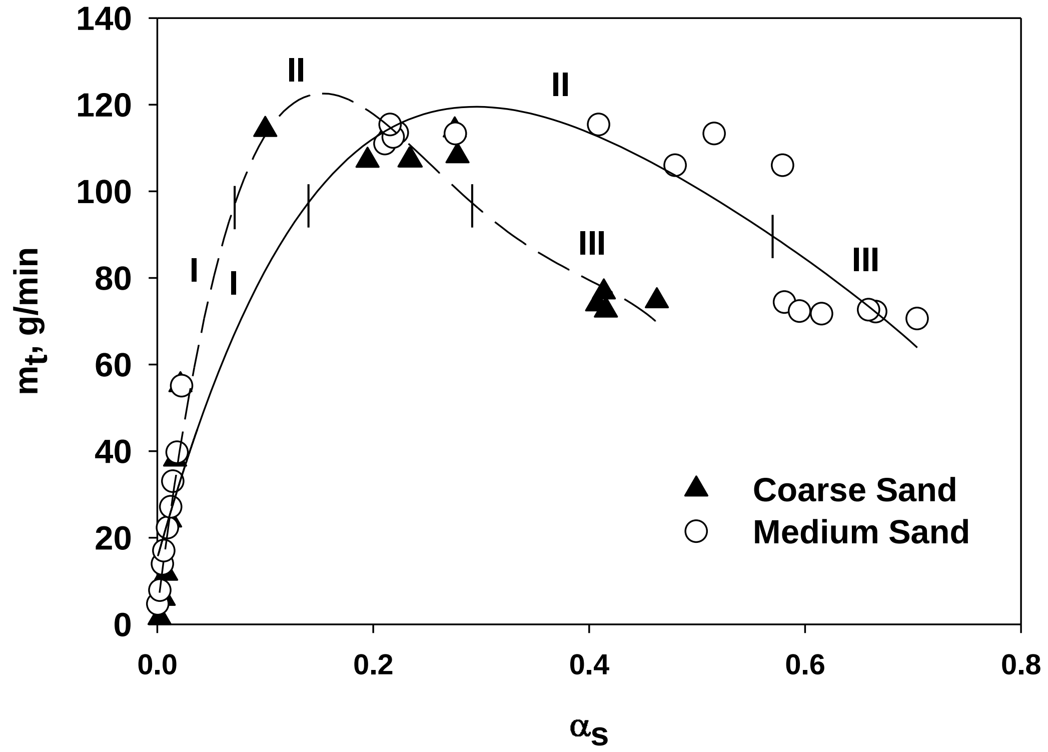


Figure : Transport rate-concentration curve for sand transport in shallow streams. Flow rate, 21.6 l/min. Labels I, II, and III represent saltation, sediment waves (stripes), and sediment waves (meanders) respectively.

Velocity vs. Concentration Measurements

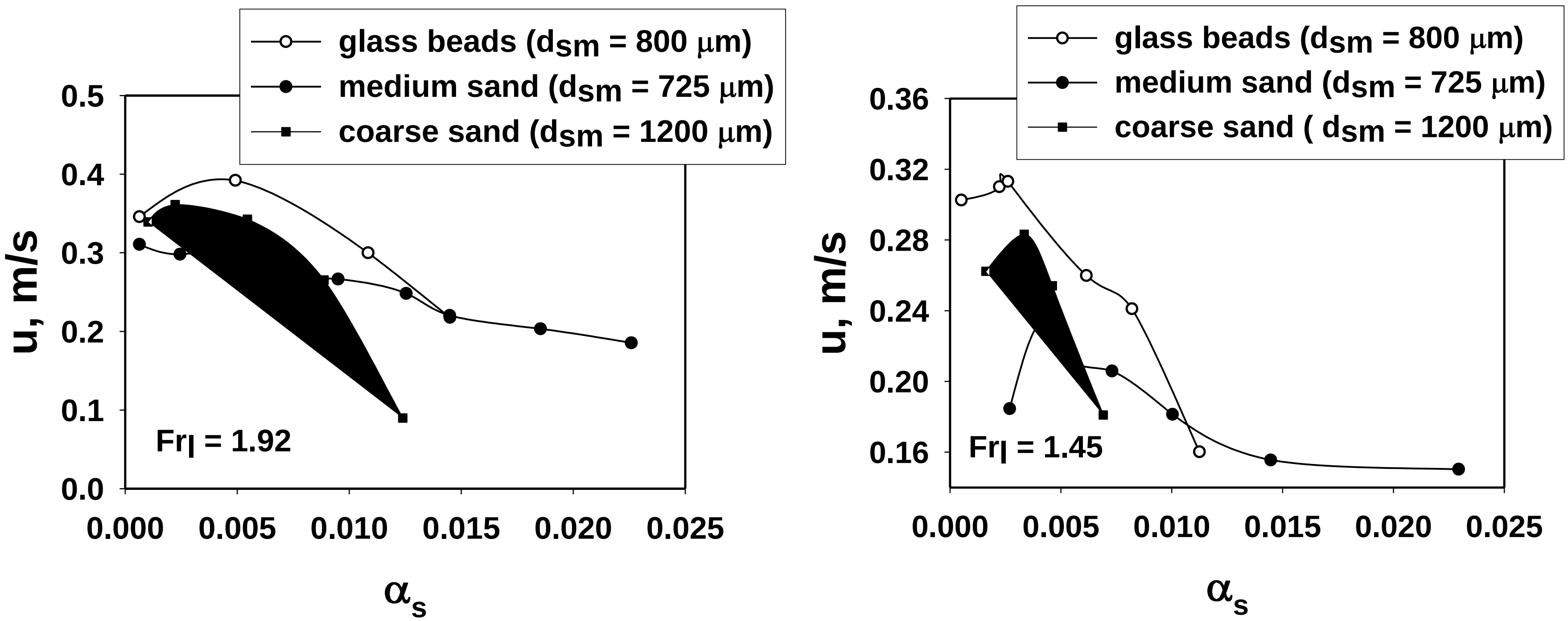


Fig : Measured velocity vs. concentration relationships for glass beads, coarse and medium sized sand for two hydraulic regimes (Fr_l = 1.92 and Fr_l = 1.45)

Analysis

Based on solutions of the conservation of mass and momentum equations. For this case with water and relative large size particles a two-layered flow system was assumed to exist. A schematic of this transport model is given:

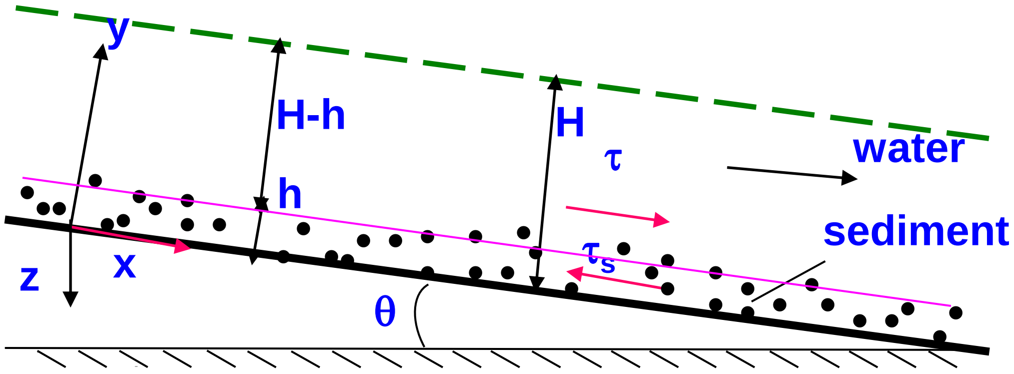


Fig: Sediment transport in water over an inclined channel

τ_s - unit width dispersive stress in the sediment
τ - unit width tractive hydro-dynamic stress on the sediment
h - saltation height
H - water depth (flow)
θ - bed slope and x,y,z are coordinates

Analysis

1. Differential equation

Sediment transport relationship of solid concentration (α) as a function of the moving spatial co-ordinate X = (x-ct)

$$3\rho_0 \frac{(u-c)^2}{\alpha^2}\left(1+\frac{1}{\alpha}\right)^{-4}\frac{d\alpha}{dX} - \frac{5h}{\rho_s d_s}\mu\gamma\alpha^{\frac{1}{2}}\frac{d\alpha}{dX} =$$

$$\frac{\rho_w hg}{\rho_s d_s}\frac{d\eta}{dX} - \rho_0 g\left(1+\frac{1}{\alpha}\right)^{-3}\sin\theta + \frac{h}{\rho_s d_s}\left(2.25\mu\gamma\alpha^{\frac{3}{2}} - \tau\right) \quad \textbf{(1)}$$

Analysis

2. Critical solid concentration

A simple criteria for the transition of grain process from saltation to strip mode is obtained from equation (1) by setting the coefficient of ($d\alpha/dX$) to zero.

Then, the critical linear solid fraction α_c is implicitly given by,

$$\frac{\alpha^6}{(1+\alpha)^2} = \frac{3\, q_m^{\,2}}{48\left(\frac{h}{d_s}\right) C_0^{\,2}\, \rho_s\, \rho_w\, \nu\, U} \qquad \textbf{(2)}$$

q_m – Sediment mass flow rate per unit width of the channel.

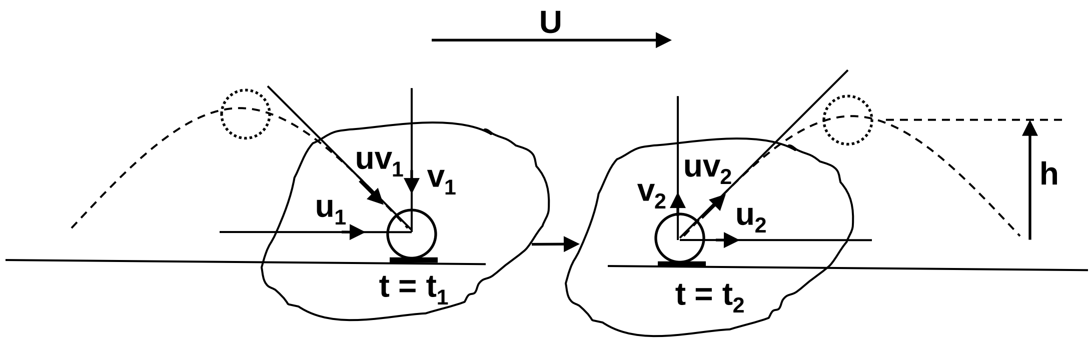
Particle velocity vs concentration

$$\mathbf{u} = \mathbf{10.7\,C_m\,C_0\,U\,\alpha^{\frac{3}{2}}}\left(h/d_s\right) \qquad \textbf{(3)}$$

u – particle velocity; C_m – Coefficient for the particle matrix (Eames et al., 2004) ; C_0 – maximum possible volumetric concentration ; U – free stream velocity of water ; α - linear particle concentration ; h – saltation height ; d_s – particle diameter

Eames, I., J. C. R. Hunt, S. E. Belcher. 2004. Inviscid mean flow through and around groups of bodies. *J. Fluid Mech.* 515: 371-389.

Boundary Effect – Conceptual Model of Particle Impact Mechanics



$$\text{Impact pulse} = \int_{t_1}^{t_2} F\,dt = m(uv_1 - uv_2) \quad \text{with}$$

$$\text{Impulse components: } I_n = 2m\sqrt{2gh} \;;\; I_t = 0.25 I_n$$

Relationship : $\dfrac{I_t}{I_n} = \dfrac{m(u_1 - u_2)}{m(v_1 - v_2)} = \dfrac{u_1 - u_2}{2v_1} = 0.25$ **Large oblique impact.**

$t = t_1$ $u_1 = u_0$ $t = t_2$ $u_2 = u_t = e_t u_0$ e_t = coefficient of restitution

Result : $h = 206.6(1 - e_t)^2 u_0^2$ **(4)**

Prasad, S.N., M.R. Suryadevara, M.J.M. Römkens. 2009. Grain transport mechanics in shallow overland flows. *Ecohydrology* 2(3): 248-256.

Boundary effect

Assume $u = (u_0+u_t)/2$, **then**

$$h = 8.26 \times 10^2 \left[(1-e_t)/(1+e_t)\right]^2 u^2 \qquad (5)$$

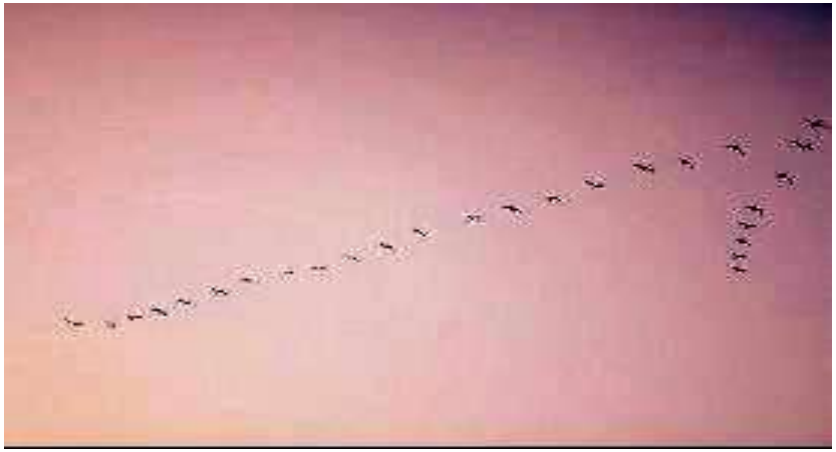
Which upon substitution in the particle velocity – linear concentration relationship yields

$$u = \frac{1.13 \text{x} 10^{-4} d_s}{C_m C_0 U \alpha^{1.5}} \left(\frac{1+e_t}{1-e_t} \right)^2 \qquad (6)$$

A number of interactions were considered : particle–fluid interaction (Bagnold dispersive pressure); particle–boundary impact (grain mechanics). Particle-particle interactions (collisions). However, not considered is the particle–particle interactions at low concentrations that leads to drag reduction. That may be important in the formation of clusters and sediment waves.

Can we in sedimentary fluid mechanics learn something from fish and birds ?

Birds migration. A location in Israel. Photo Courtesy : C. I. Cohen.



Birds flock Vee formation. Location in Denmark. Photo Courtesy : A. Filippone.

Tropical under water life. Photo Courtesy : Kevin Crane.

Saltating coarse sand

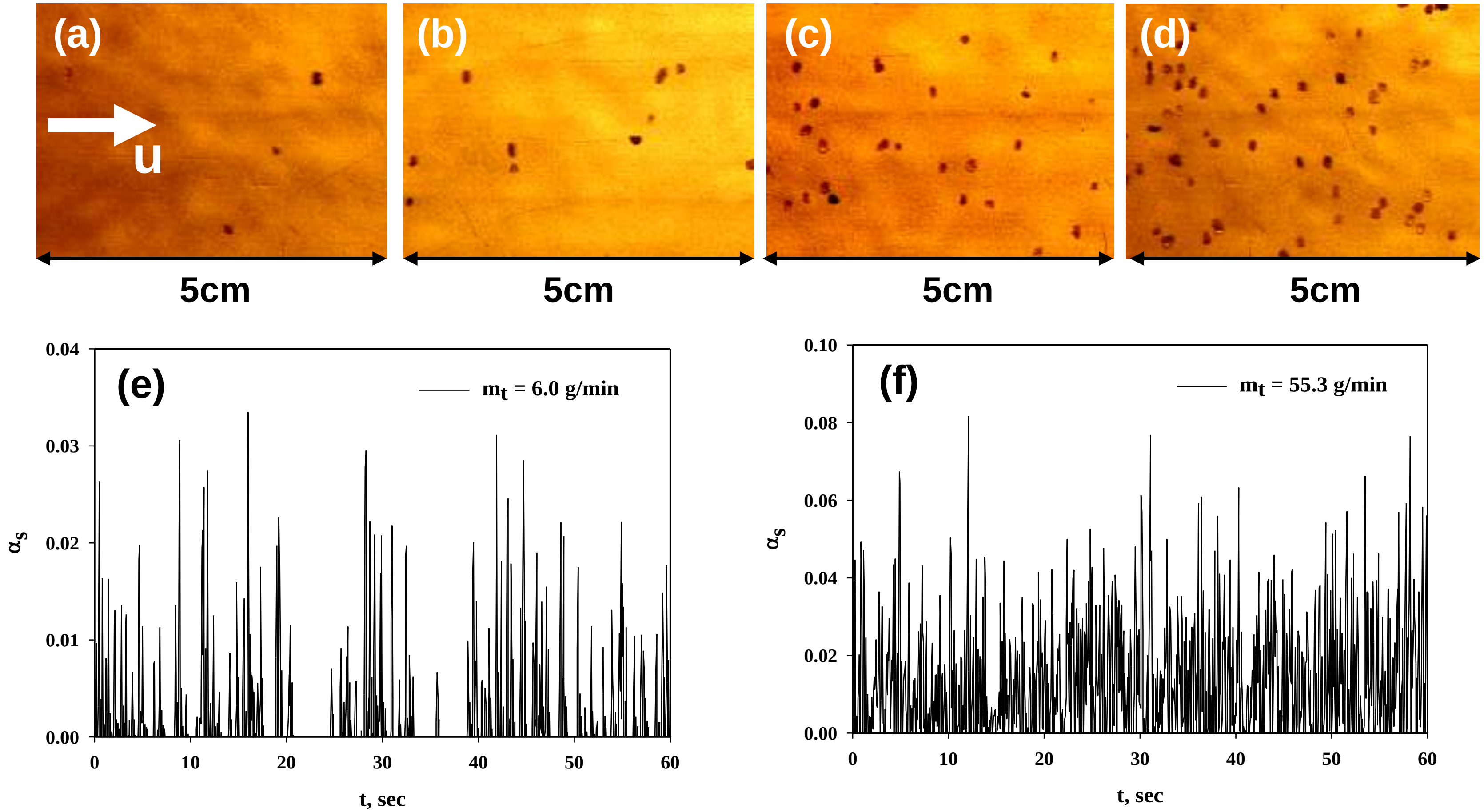
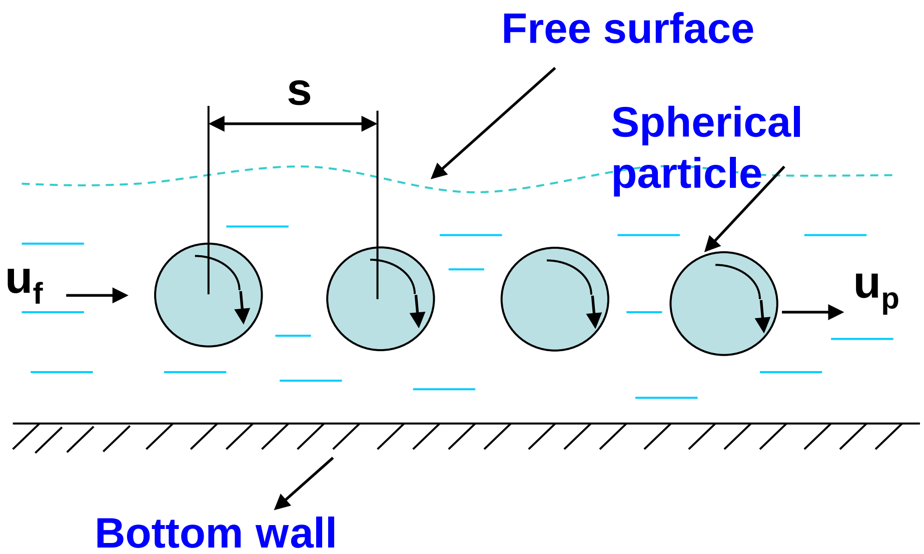


Fig : Saltating flow of coarse sand particles (d_{smean} = 1200 μm) with increasing sediment transport rates and a water flow rate of 21.6 l/min (Fr_l = 1.92). Typical photographs [figs (a)-(d)] of saltating grains for four transport rates m_t = 6.0 g/min, 11.8 g/min, 38.1 g/min & 55.3 g/min respectively. Figs (e) & (f) represent changes in the planar concentration with time for figs. (a) & (d), respectively.

Drag quantification



$$u_0 = u_f - u_p \qquad (1)$$

u_0 - slip velocity, u_f - fluid velocity at the elevation of the particle. u_p – particle velocity.

Drag force on a particle,

$$F_D = C_D u_0^2 \qquad (2)$$

Where C_D is the coefficient of drag.

Drag quantification

Osceen's expression for stream function to account for the inertial effect of fluid motion (This may be of importance for the Reynolds number in the wake region away from the particle center)

$$\psi = \frac{u_0 d_s^{\,2}}{16}(r-1)^2 \sin^2\theta \left[\left(1+\frac{3R}{8}\right)(2+r)^{-1} - \frac{3R}{8}\left(2+\frac{1}{r}+\frac{1}{r^2}\right)\cos\theta \right] \qquad (3)$$

Drag force following correction :

$$D_* = 3\pi\mu u_0 d_s \qquad (4)$$

Where, μ - Coefficient of viscosity

Drag quantification

Velocity distribution in wake region

$$u_r = \frac{1}{r^2.\sin\theta}.\frac{\partial\psi}{\partial\theta} \approx \frac{u_0 d_s}{4r}\left(3 - \frac{d_s^{\ 2}}{4r^2}\right) \qquad (5)$$

Drag of lead particle

$$F_{D1} = C_D u_0^{\ 2} \qquad (6)$$

Drag of following particle – Net drag effect

$$F_{D2} = C_D\left(u_0^{\ 2} - u_0^{\ *2}\right) \qquad (7)$$

$\alpha << 1$

$$F_{D2} = C_D u_0^{\ 2}\left(1 - \frac{9}{16}\alpha^2\right) \qquad (8)$$

Drag quantification

Net drag effect for multiple particles

On the third particle

$$F_{D3} = C_D u_0^2 \left[1 - \frac{9}{16} \left(1 + \frac{1}{2^2} \right) \alpha^2 \right] \quad (9)$$

On the Nth particle

$$F_{DN} = C_D u_0^2 \left[1 - \frac{9}{16} \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \frac{1}{N^2} \right) \alpha^2 \right] \quad (10)$$

Net drag on uniform spaced particles

$$F_{DN} = C_D u_0^2 \left[1 - \frac{9}{16} \frac{\pi^2}{6} \alpha^2 \right] \quad (11)$$

Drag reduction coefficient

$$k = \frac{F_{DN}}{C_D u_0^2} = \left(1 - \frac{3\pi^2}{32} \alpha^2 \right) \quad (12)$$

Conclusion

All evidence points out that sedimentary fluid mechanics seems to play a much larger role in sediment transport problems than so far has been assumed.